FOUR COMPUTING

OPTIMIZATION

Social Choice

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1. Social choice

The discipline of social choice originates from objective probability used in jurisdiction and justice and as pioneered by French mathematicians Borda[1] and de Condorcet[2]. The balance between individual liberty and societal authority was related to the risk of innocent citizens being wrongly convicted and punished for crime. Social justice as well as social order required that particular risk to be minimized.

Condorcet argued that judicial tribunals could manage probabilities and errors, taking in to account also some minimum required plurality to guarantee the probability. Uncertainty based voting and related consensus schemas are then just behind the corner, and are the historical prerequisites also for choice theory.

Arrow[3] focused on individual values and ranking, together with impossibility theorems, and dealt with individual preferences and choice processes. Probability is not a prerequisite, and choice is viewed rather as operators and functions, and properties about them. There are no counterparts in probability theory for these concepts. From a logical point of view, Arrow uses implicitly underlying signatures, even if they are never formalized, and since they aren't formalized, it is never seen that that these choice functions indeed could have been integrated into a logical framework. Neumann and Morgenstern[4] follow Arrow's mathematical tradition in their success stories of economic and social sciences, also without ending up in any logical framework.

Our standpoint is that social choice functions must identify the difference between ‘we choose' and ‘our choice', the former being the operation of choosing, the latter being the result of that operation. We view this from a signature point of view, i.e., using formalism involving signatures and their algebras in a generalized universal algebra setting.

Going beyond the distinction between choosing and choice, and entering rationality of choice, Mill[5] said that behaviour is based on custom more than rationality. Custom is clearly based on particular algebras acting as models and used in logical satisfaction, whereas rationality is based on representable sentences interrelated by logical entailment. Consensus reaching[6] embraces a dynamic situation of aggregated choice, where individual preferences change within a consensus reaching mechanism. This opens up interesting perspectives as consensus reaching in our substitution model for social choice now also reaches the level of \`choosing', i.e. consensus is reached either on \`choice' level including dynamics and negotiation for the \`choosing' level, or can even be a stronger consensus on \`choosing' levels as well.

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[1] J. C. Borda, *Memoir sur les elections au scrutin*, Histoire de l'Academie Royale des Sciences, Paris, 1781.
[2] M. de Condorcet, *Essai sur l'application de l'analyse a des decisions rendues a la pluralite des voix*, L'Imprimerie Royale, Paris, 1785.
[3] K. J. Arrow, *Social Choice and Individual Values*, Wiley, New York, 1951.
[4] J. von Neumann, O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, 3rd Ed., 1953.
[5] J. S. Mill, *Principles of Political Economy*, Seventh Edition, London; Longmans, Green and Co., 1948 (First Edition, 1909).
[6] P. Eklund, A. Rusinowska, H. de Swart, *A consensus model of political decision-making*, Annals of Operations Research **158** (2008), 5-20.

2. Preferences and multiple-criteria decisions

Let X be a set of alternatives[7], and write $\mathrm{card}(X)$ for the (finite) cardinality[8] of X. Further, let L be a distributive lattice[9]. A customer is a mapping[10] $\alpha: X \to L$, and a customer classifier is a mapping $\kappa: L^X \to L$. Similarly, an offer[11] is a mapping $\beta: X \to L$, and an offer classifier is a mapping $\lambda: L^X \to L$. Let $A \subseteq L^X$ be the set of target customers, and $B \subseteq L^X$ the set of target offers.

For an offer β and a customer α, $\min\{\alpha(x), \beta(x)\}$ is the customer acceptance degree for the product $x \in X$ with respect to the offer, and $\max_{x \in X} \min\{\alpha(x), \beta(x)\}$ is the overall acceptance degree with respect to the offer.

In general we want to maximize the number of customers, so we want to find that particular offer which maximizes the number of customers accepting that offer, i.e. we want to determine

$$\max_{\beta \in B} \sum_{\alpha \in A} \max_{x \in X} \min\{\alpha(x), \beta(x)\}.$$

If α and β are crisp and represented as bit arrays, then $\max_{x \in X} \min\{\alpha(x), \beta(x)\}$ corresponds to an appropriate conjunction of α and β.

Note that

$$\max_{\beta \in B} \max_{x \in X} \sum_{\alpha \in A} \min\{\alpha(x), \beta(x)\} \leq \max_{\beta \in B} \sum_{\alpha \in A} \max_{x \in X} \min\{\alpha(x), \beta(x)\}.$$

and

$$\max_{\beta \in B} \sum_{\alpha \in A} \max_{x \in X} \min\{\alpha(x), \beta(x)\} \leq \sum_{\alpha \in A} \max_{\beta \in B} \max_{x \in X} \min\{\alpha(x), \beta(x)\}.$$

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[7] It could e.g. be a set of non-durable consumer products, or product groups, for food, clothing, and cosmetics, or durable products like electronics, house-hold and furniture.
[8] Number of distinct point in the set.
[9] Often we use L=[0,1], or even L={0,1}. The lattice is usually totally ordered.
[10] This mapping is frequently called a *fuzzy set*, or *many-valued set*.
[11] Offers β are typically crisp sets, so that $\sum_{x \in X} \beta(x)$ is the numbers of alternative products offered to customers.